PMM U.S.S.R.,Vol.47,No.3,pp.366-370,1984
Printed in Great Britain

0021-8928/84 $10.00+0.00

UDC 533.6.011

TWO-DIMENSIONAL DISCHARGE INTO VACUUM OF A MOVING INHOMOGENEOUS GAS*

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Two-dimensional nonisotropic flows of perfect polytropic gas are considered, which occur when the smooth surface that separates the given flow of gas from vacuum is instantly removed. Solution of the problem of such discontinuity disintegration is constructed in the space of special variables in the form of convergent characteristic series. On the basis of investigation of the series convergence region it is proved that gas particles at the boundary with vacuum continue moving for some time each along its straight path at its constant velocity. Next, the case of continuous contiguity of gas through the smooth free surface to vacuum is considered. It is shown that up to the instant of occurrence of an infinite gradient at the free boundary, or up to the instant of local gas focusing, the derived law of motion of the free boundary remains valid. This may be used as the boundary condition in numerical solution of problems on discharge of perfect gas into vacuum. A system of transport equations is obtained and investigated, which defines the behavior of the gradient of gas dynamic parameters at the boundary with vacuum.

1. Let some cylindrical surface Γ be specified at instant of time $t = 0$ in the space (x, y, z). Its generatrices are parallel to axis Oz and the directrix in the plane xOy is specified parametrically $x = x(\xi)$, $y = y(\xi)$, where ξ is a parameter, and the functions themselves are assumed locally analytic in some neighborhood of point $(x_0 = x(\xi_0),\ y_0 = y(\xi_0))$. On one side of the surface Γ is the vacuum, and on the other there are some locally analytic distribution of parameters of a perfect polytropic gas, viz, the gas velocity $\mathbf{U} = \mathbf{U}_0(x, y)$, its entropy $S = S_0(x, y)$, $\sigma = \sigma_0(x, y)$ some functions related to density ρ by the relation $\sigma = \rho^{(\gamma-1)/2}$ and $(\gamma > 1)$ the polytropic exponent of gas. The specified distributions are such that on surface Γ the speed of sound $c = S\sigma$ is greater than zero, and the projection of vector $\mathbf{U}_0$ on the Oz axis is zero. At instant $t = 0$ the surface Γ momentarily disintegrates, and at $t > 0$ the discharge of gas into vacuum takes place. We assume that as the result of discontinuity disintegration the two-dimensional flow is fairly smooth, and is bounded on one side by the surface of a weak discontinuity Γ_1 and on the other, by the free surface Γ_0.

The law of motion of surface Γ_1 and the gasdynamic parameters on it, by virtue of the theorem of Cauchy–Kovalevskaia, are uniquely determined by the distribution at $t = 0$ /1/. The position of the free surface Γ_0 which is the boundary between the gas and vacuum $(\sigma|_{\Gamma_0} = 0)$, at various instants of time is not known generally, a priori. As the flow is two-dimensional, we shall identify subsequently the surfaces Γ, Γ_0, and Γ_1 by respective curves in the plane xOy.

Problems close to the one stated here were considered earlier. In the flow of homogeneous gas from a slanted wall the inhomogeneous part of the flow was contructed in /2/ for special values of γ in the class of self-similar double waves. Two- and three-dimensional flows of gas were constructed in /3/, using characteristic series, and in /4/ two-dimensional flows of gas were obtained in some one-sided neighborhood of Γ_1. On the other side of the weak discontinuity surface, either a homogeneous quiescence /3/ or a region of simple wave /4/ were obtained. Investigation of several terms of series for an approximate description of flow in the free boundary neighborhood led to a flow close to one-dimensional /3/, and in /4/ to a flow close to the constructed in /2/. The question whether the free boundary is included in the series convergence region was not investigated in /3,4/. The disintegration of an arbitrary discontinuity on a curvilinear surface was considered in /5/, when on both sides of the discontinuity surface the density is greater than zero. A flow orginates at the collapse of one-dimensional cavity was investigated in /6/. It was proved that when $1 < \gamma < 3$ Γ_0 moves for some time $0 \leqslant t < t_*$ at constant velocity, while time t_* coincides with the instant of an infinite gradient formation on Γ_0. The transport equation that controls the behavior of gas parameter gradients on Γ_0 is investigated.

The present paper extends the results of /6/ to the case of arbitrary two-dimensional nonisotropic flows.

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*Prikl.Matem.Mekhan.,Vol.47,No.3,pp.433-439,1983

The motion of a perfect polytropic gas is defined by the system of equations /7/

$$\sigma_t + \mathbf{U}\,\mathrm{grad}\,\sigma + \frac{\gamma-1}{2}\sigma\,\mathrm{div}\,\mathbf{U} = 0$$

$$\mathbf{U}_t + \mathrm{rot}\,\mathbf{U}\times\mathbf{U} + \frac{1}{2}\mathrm{grad}\,\mathbf{U}^2 + \frac{S^2}{\gamma-1}\mathrm{grad}\,\sigma^2 + \frac{\sigma^2}{\gamma}\mathrm{grad}\,S^2 = 0$$

$$S_t + \mathbf{U}\,\mathrm{grad}\,S = 0$$

The equation of state is taken in the form $p = A(S)\rho^\gamma/\gamma$, p is the pressure and $A(S) > 0$. In the considered here system and subsequently the entropy is denoted by $\sqrt{A(S)}$.

In the system we pass from variables x, y to η, ξ, and from each point of curve Γ draw a straight line whose directional vector is

$$\mathbf{U}_* = \mathbf{U}_0 + \frac{2}{\gamma-1}S_0\sigma_0\mathbf{n}$$

where $(\mathbf{n} = \mathbf{n}(x, y)$ is the unit vector of the normal to Γ. These lines are taken as the coordinate lines $\xi = \mathrm{const}$. As the second coordinate lines we take the curves orthogonal to the constructed bundle of straight lines $\eta = \mathrm{const}$. In particular the curve that passes through point (x_0, y_0) is taken as the axis $\eta = 0$. The one-to-one relation of such transition from (x, y) to (η, ξ) is ensured by the condition $\mathbf{U}_*\mathbf{n} \neq 0$ in which vectors U_* are not tangent to line Γ. Subsequently this condition will be taken as satisfied. The possibility of passing from (x, y) to (η, ξ) is ensured by the existence of a unique locally analytic solution of the respective Cauchy problem for the ordinary differential equation that defines the curve $\eta = 0$. After such substitution, the input system becomes /7/

$$\sigma_t + u\sigma_\eta + \frac{1}{H_2}\left\{v\sigma_\xi + \frac{\gamma-1}{2}\sigma[H_2u_\eta + B(\xi)u + v_\xi]\right\} = 0 \tag{1.1}$$

$$u_t - \frac{1}{H_2}v[B(\xi)v - u_\xi] + uu_\eta + \frac{2}{\gamma-1}S^2\sigma\sigma_\eta + \frac{2}{\gamma}\sigma^2SS_\eta = 0$$

$$v_t + \frac{1}{H_2}\left\{u[H_2v_\eta + B(\xi)v] + vv_\xi + \frac{2}{\gamma-1}S^2\sigma\sigma_\xi + \frac{2}{\gamma}\sigma^2SS_\xi\right\} = 0$$

$$S_t + uS_\eta + \frac{1}{H_2}vS_\xi = 0, \quad H_2 = A(\xi) + \eta B(\xi)$$

$$A(\xi) = [\varphi_1'^2(\xi) + \varphi_2'^2(\xi)]^{1/2}, \quad B(\xi) = [\varphi_1''(\xi)\varphi_2'(\xi) - \varphi_2''(\xi)\varphi_1'(\xi)]/A^2(\xi)$$

where u, v are projections of vector $\mathbf{U}$ on the axis η, ξ, respectively. Using $\varphi_1(\xi), \varphi_2(\xi)$, the curve $\eta = 0$ is defined parametrically $x = \varphi_1(\xi)$, $y = \varphi_2(\xi)$.

Note that $A(\xi_0) \neq 0$, i.e., (x_0, y_0) is not a singular point of line $\eta = 0$. In the new system of coordinates curve Γ is specified by some locally analytic function $\eta = \eta_{00}(\xi)$, $\mathbf{U}_* = \{u_*(\xi), 0\}$, and the second component of vector $\mathbf{U}_0|_\Gamma$ is equal

$$v_0 = -\frac{2}{\gamma-1}S_0\sigma_0\cos(\mathbf{n}, \boldsymbol{\tau}) = \frac{2}{\gamma-1}S_0\sigma_0\frac{\eta_{00}'(\xi)}{A_2(\xi)}$$

$$A_2(\xi) = [\eta_{00}'^2(\xi) + A_1^2(\xi)]^{1/2}, A_1(\xi) = A(\xi) + \eta_{00}(\xi)B(\xi)$$

where $\boldsymbol{\tau}$ is the unit vector tangent to line $\eta = \mathrm{const}$, that passes through point $(x(\xi), y(\xi))$ of line Γ.

Since the unknown solution at the initial instant of time has unbounded derivatives with respect to η, we make one more substitution of variables. As independent variables we take σ, ξ, t, and η, u, v, S as the unknown functions. The Jacobian of such transformation is $J = \eta_\sigma$. It is zero, if in space (η, ξ, t) an infinite gradient occurs and, then, in space (σ, ξ, t) the solution has no singularity. The opposite takes place when $J = \infty$: in space (σ, ξ, t) the solution has an infinite gradient, and singularity in space (η, ξ, t) is absent. System (1.1) becomes

$$H_2(u - \eta_t) - v\eta_\xi + \frac{\gamma-1}{2}\sigma[H_2u_\sigma + B(\xi)u\eta_\sigma + v_\xi\eta_\sigma - v_\sigma\eta_\xi] = 0 \tag{1.2}$$

$$\eta_\sigma H_2u_t + H_2(u - \eta_t)u_\sigma - B(\xi)v^2\eta_\sigma + vu_\xi\eta_\sigma - vu_\sigma\eta_\xi +$$
$$2H_2S\sigma\left(\frac{S}{\gamma-1} + \frac{\sigma}{\gamma}S_\sigma\right) = 0$$

$$\eta_\sigma H_2v_t + H_2(u - \eta_t)v_\sigma + B(\xi)uv\eta_\sigma + vv_\xi\eta_\sigma - vv_\sigma\eta_\xi -$$
$$2S\sigma\left(\frac{S}{\gamma-1}\eta_\xi - \frac{\sigma}{\gamma}S_\xi\eta_\sigma + \frac{\sigma}{\gamma}S_\sigma\eta_\xi\right) = 0$$

$$\eta_\sigma H_2S_t + H_2(u - \eta_t)S_\sigma + vS_\xi\eta_\sigma - vS_\sigma\eta_\xi = 0$$

Theorem 1. When $0 \leqslant t \leqslant t_0, t_0 > 0$, then in some neighborhood of line Γ_1 there exists a unique locally analytic solution of system (1.2) that corresponds to the input problem of discontinuity disintegration.

To prove it, theorem 1 is reduced to the respective analog of the Cauchy–Kovalevskaia theorem /8/. In this case the Cauchy data for system (1.2) are specified on the characteristic of Γ_1. To ensure the uniqueness of solution of such problem it is necessary /8/ to specify one boundary condition, since Γ_1 is a characteristic of multiplicity of one $(c|_{\Gamma_1} > 0)$. This condition is provided by the relation $\eta(0, \sigma, \xi) = \eta_{00}(\xi)$. Indeed, if in the region of flow obtained as the result of discontinuty disintegration we consider surface $\eta = \eta(t, \sigma, \xi)$, then as $t \to +0$ that surface, irrespective of the dependence on σ, becomes surface $\eta = \eta_{00}(\xi)$.

To define more precisely the region of existence of solution of that characteristic problem of Cauchy it is necessary that it is represented in the form of series

$$\mathbf{f}(t, \sigma, \xi) = \sum_{k=0}^{\infty} \mathbf{f}_k(\sigma, \xi) \frac{t^k}{k!}, \qquad \mathbf{f} = \{\eta, u, v, S\} \tag{1.3}$$

The coefficients $\mathbf{f}_k$ are obtained, as in /6/, in a recurrent manner in successive differentiation of system (1.2) with respect to t. Then u_k, v_k, S_k are obtained from the solution of a system of ordinary differential equations in which ξ is a parameter. Then η_{k+1} is found from an algebraic equation. These systems and equations are not presented here in view of their unwieldiness. The result is of the form

$$u_0 = -\frac{2}{\gamma-1} S_{00}(\xi) \frac{A_1(\xi)}{A_2(\xi)} \sigma + u_{00}(\xi),$$

$$v_0 = \frac{2}{\gamma-1} S_{00}(\xi) \frac{\eta_{00}'(\xi)}{A_2(\xi)} \sigma + v_{00}(\xi)$$

$$S_0 = S_{00}(\xi), \quad \eta_1 = u_0 - \frac{1}{A_1(\xi)} \left\{ \left[\frac{2}{\gamma-1} S_{00}(\xi) \eta_{00}'(\xi) + A_2^2(\xi) \right] \times \frac{\sigma}{A_2(\xi)} + v_{00}(\xi) \eta_{00}'(\xi) \right\}$$

$$u_k = u_{k0}(\xi) \sigma^{k\alpha} + \eta_{00}'(\xi) v_{k0}(\xi) \frac{\sigma^{2k\alpha}}{A_1(\xi)} + F_{1k}$$

$$v_k = -\eta_{00}'(\xi) u_{k0}(\xi) \frac{\sigma^{k\alpha}}{A_1(\xi)} + v_{k0}(\xi) \sigma^{2k\alpha} + F_{2k}$$

$$S_k = S_{k0}(\xi) \sigma^{2k\alpha} + F_{3k}, \quad \eta_{k+1} = F_{4k}, \quad \alpha = \frac{\gamma+1}{2(\gamma-1)}$$

Functions F_{ik} $(1 \leqslant i \leqslant 4)$ depend in a known manner on σ, ξ and on preceding coefficients of series (1.3) (in view of its unwieldiness the specific form of F_{ik} is also not presented). The arbitrary functions u_{k0}, v_{k0}, S_{k0}, and $k \geqslant 0$ appear here, as the result of integration of systems of differential equations. They are to be selected so as to match series (1.3) with the gasdynamic parameters on Γ_1. Since $\sigma|_{\Gamma_1} > 0$, all w_{k0}, v_{k0}, S_{k0} are uniquely determined. In particular, $v_{00}(\xi) = 0$, $S_{00}(\xi) = S_0(x, y)|_{\Gamma}$.

Lemma. When $1 < \gamma < 3$, the coefficients $u_k, v_k, S_k, \eta_{k+1}$ for $k \geqslant 1$ contain the multiplier σ and are polynomials of $\sigma, \sigma\ln\sigma, \sigma^\lambda, 0 \leqslant \lambda \leqslant A_3 k, A_3 = \text{const} > 0$. The coefficients of polynomials depend on ξ. When $\gamma \geqslant 3$, the coefficients of series contain $\sigma^{-1}, \ln\sigma$.

Proof of the lemma is similar to the respective proof in /6/.

The lemma is used for proving the following theorem.

Theorem 2. When $1 < \gamma < 3$ and $0 \leqslant t \leqslant t_0$, the convergence region of series (1.3) and of $\mathbf{f}_t, \mathbf{f}_\sigma, \mathbf{f}_\xi$ is extended over the whole region from Γ_1 to Γ_0 inclusive. The position of Γ_0 is defined by the relation $\eta = \eta_{00}(\xi) + u_*(\xi)t$, and on the surface Γ_0 we have $\mathbf{U} = \mathbf{U}_*(\xi)$, $S = S_{00}(\xi)$.

Thus each particle of gas on the free surface, after the discontinuity disintegration, maintains its entropy, and moves along its straight line at its constant velocity, as if at each point of surface Γ its own disintegration of a plane discontinuity had taken place.

Series (1.3) is a functional series that is analytic only in some neighborhood of surface Γ_1: for any $\gamma > 1$ an m can be found such that the series for the derivative $\partial^m \mathbf{f}/\partial\sigma^m$ contain σ^{-1} and $\ln\sigma$. As $\sigma \to 0$, the series for such derivative is certainly divergent, and the next following term is greater than the preceding ones. When $1 < \gamma < 3$, this m is greater than unity, and the series required for satisfying system (1.2) with $\sigma = 0$ are convergent. When $\gamma = 3$ $m = 1$ or $\gamma > 3$ and $m = 0$, and on the basis series consideration it is impossible to make any exact conclusions about the motion of Γ_0. In the case of cylindrical and spherical symmetry we have the alternatives: either $\eta_\sigma|_{\Gamma_0} = 0$ and Γ_0 is accelerated or $\eta_\sigma|_{\Gamma_0} \neq 0$

and Γ_0 moves at constant velocity. Hence, if the flow resulting from discontinuity disintegration satisfies the relation

$$\eta_{\sigma t}(0,0) = \eta_{t\sigma}(0,0)$$

then for all $\gamma > 1$ the surface Γ_0 moves for some time at constant velocity

$$U_* = U_0(x_0) \pm \frac{2}{\gamma-1} S_0(x_0)\sigma_0(x_0)$$

where the upper sign corresponds to a dispersion of gas, the lower, to collapse. In proving this fact only the value of $\eta_t(\sigma, 0)$ is used, the total series are unnecessary.

2. Having obtained the solution of the problem of discontinuity disintegration at small t, we shall clarify up to what instants of time the obtained law of propagation of Γ_0 is valid. For this in system (1.1) instead of the variable η we take the variable $\chi = \eta - \eta_{00}(\xi) - u_*(\xi)t$, i.e. the free surface Γ_0 is taken as the new coordinate plane.

If at $t = t_0$ in some neighborhood of surface $\chi = 0$ a locally analytic distribution of gas parameters is specified

$$\begin{aligned} &\sigma(t_0, \chi, \xi) = \sigma_{01}(\chi, \xi), \ \sigma_{01}(0, \xi) = 0 \\ &\mathbf{U}(t_0, \chi, \xi) = \mathbf{U}_{01}(x, \xi), \ \mathbf{U}_{01}(0, \xi) = \mathbf{U}_*(\xi) \\ &S(t_0, \chi, \xi) = S_{01}(\chi, \xi), \ S_{01}(0, \xi) = S_{00}(\xi) \end{aligned} \tag{2.1}$$

then the obtained Cauchy problem for system (1.1) in variables χ, ξ, t has in conformity with the Cauchy–Kovalevskaia theorem a unique locally analytical solution. To clarify the law of motion of Γ_0 we consider, as in /6/, the Cauchy problem with data on the surface $\chi = 0$: $\sigma = 0$, $\mathbf{U} = \mathbf{U}_*(\xi)$, $S = S_{00}(\xi)$. This is the characteristic Cauchy problem which at $t_0 \leqslant t < t_*$ has a unique locally analytic solution with specified supplementary condition /8/. Such conditions for the considered here problem are relations (2.1). From the uniqueness of these solutions of the Cauchy problem follows their agreement. Hence for $t_0 \leqslant t < t_*$ Γ_0 moves in conformity with the same law $\eta = \eta_{00}(\xi) + u_*(\xi)t$.

As in /6/, the investigation of the existence of region of solution of the characteristic Cauchy problem with data on surface $\chi = 0$ leads to the following. The instant of time t_* is the instant of appearance of solution singularities in the system of transport equations

$$\begin{aligned} &\sigma_{1t} + [(\gamma+1)Z + (\gamma-1)Y_1]\sigma_1/2 = 0 \\ &u_{1t} + Zu_1 + u_*'(\xi)v_1/H_{20} + X = 0 \\ &v_{1t} + (Z + Y_1)v_1 - Y_2X/H_{20} = 0 \\ &S_{1t} + ZS_1 + S_{00}'(\xi)v_1/H_{20} = 0 \end{aligned} \tag{2.2}$$

where

$$\begin{aligned} &Z = u_1 - Y_2v_1/H_{20}, \quad X = 2S_{00}^2(\xi)\sigma_1^2/(\gamma-1) \\ &Y_1 = u_*(\xi)B(\xi)/H_{20}, \ Y_2 = \eta_{00}'(\xi) + u_*'(\xi)t \\ &H_{20} = A(\xi) + [\eta_{00}(\xi) + u_*(\xi)t]B(\xi) \end{aligned}$$

System (2.2) defines the behavior of the first derivative of gasdynamic parameters on surface Γ_0

$$g_1(t,\xi) = \partial g/\partial\chi\,|_{\chi=0}, \ g = \{\sigma, u, v, S\}$$

and is, in fact, a system of ordinary differential equations in which ξ is a parameter. Note that $\partial/\partial\chi = \partial/\partial\eta$. The initial data for system (2.2) are taken from conditions (2.1), and the obtained Cauchy problem has a unique locally analytic solution.

The first three equations of system (2.2) may be solved independently of the last one which is linear in S_1. Hence the anisentropy of flow does not affect the instant of singularity formation on surface Γ_0. Introducing the new unknown function

$$y_1 = \exp\int_{t_0}^{t} u_1\,dt, \quad y_2 = \exp\left[-\int_{t_0}^{t}\frac{Y_2v_1}{H_{20}}\,dt\right]$$

we eliminate σ_1 form the second and third equations, i.e. obtain a system of two equations for y_1, y_2.

When the gas is homogeneous and at rest at $t = 0$ (i.e. the non-one-dimensionality of the generated flow is due only to the non-one-dimensionality of surface Γ), the solution of system (2.2) reduces to the solution of the single ordinary differential equation

$$[1 \pm 2k(\xi)t/(\gamma-1)]^{(\nu-1)}y_1^{\nu}y_{1tt} = C_1(\xi)$$

with initial conditions

$$y_1(t_0, \xi) = 1, \; y_{1t}(t_0, \xi) = \partial u_{01}/\partial \chi|_{\chi=0}$$

This equation is in fact a transport equation for the case of cylindrical symmetry /6/. Functions σ_1, v_1, S_1 are explicitly expressed in terms of y_1. The sign in front of $k(\xi)$, the curvature of line Γ at point ξ, is selected depending on whether the radius of curvature of line Γ_0 increases or diminishes with the increase of t.

The infinite gradient first appears on Γ_0 on that ray $\xi = \xi_1$ on which at $t = 0$ the maximum curvature is attained, under the condition that the curvature of Γ_0 on that ray increases as t increases. For $\gamma \leqslant \gamma_* = 2$ the instant $t_* = (\gamma - 1)/[2k(\xi_1)]$ is the instant of focusing of the free boundary part that adjoins the ray $\xi = \xi_1$, i.e. the instant of lens formation on Γ_0. Owing to the focusing of gas in the lens corner, the absolute velocity at that point becomes greater than $|u_*(\xi_1)|$. When $\gamma > \gamma_*$ the infinite gradient arises on the same ray $\xi = \xi_1$, but this takes place prior to the instant of lens formation, i.e. Γ_0 is still smooth. The dependence $t_* = t_*(\gamma)$ is given in /6/.

Of course, after the formation of singularities mentioned above, the use of the derived solutions in the neighborhood of such points is no longer possible, and it is necessary to solve new problems. These problems in one-dimensional approximation are: that of formation at the center of symmetry of a shock wave and its motion away from that center; the second problem is either to construct a flow with infinite gradient on Γ_0, or solve a new one of discontinuity disintegration. If the radius of curvature at all points of Γ_0 increases with increasing t, then the infinite gradient first occurs in the flow on Γ_1 at the instant of its local focusing.

The solutions constructed in explicit form, thus, solve the question of stability of the respective non-unidimensional flow. For a given inhomogeneous distribution of gas parameters at $t = 0$ and a non-unidimensional Γ it is possible to indicate beforehand what and where are the singularities that arise in the flow, and what are the qualitative effects to which they lead.

The proved theorems at large t are valid only in certain neighborhoods of surfaces Γ_0 and Γ_1. They are not, however, taking into account the possibility of singularities formation in the flow region middle part. Weak discontinuities cannot occur there, and it is necessary to watch for the appearance in the middle part of the flow region of infinite gradients. If a shock wave arrives on the surface of Γ_0, then as proved, this new discontinuity disintegrates with constant velocity along the new rays (generally for each ray its proper velocity). Further motion of Γ_0 and the values of gasdynamic parameters on it are uniquely determined by the arriving discontinuity, and this may be used as the boundary condition for the numerical solution of problems of perfect gas discharge into vacuum.

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Translated by J.J.D.